

## PHYSICS 3300 Einstein Solid Exercises

Consider two Einstein solids A and B that can exchange energy with one another, but are isolated from their surroundings. That is, the two systems are surrounded by insulating, rigid, and impermeable outer walls and are separated from each other by a conducting, rigid, and impermeable wall. You will write an Excel spreadsheet that counts the number of ways that the energy can be distributed between the two systems for given values of  $N_A$ , the number of oscillators in system A, and  $N_B$ , the number in system B.

1. Suppose that  $N_A = 2$ ,  $N_B = 2$  and initially  $E_A = 5$  and  $E_B = 1$ . What is the initial number of microstates for the composite system? The internal constraint is then removed so that the two subsystems can exchange energy. Determine the probability  $P(E_A)$  that system A has energy  $E_A$ , and the most probable energy of system A.

What is the total number of microstates after the internal constraint has been removed? Discuss the qualitative dependence of  $P(E_A)$  on the energy  $E_A$ . Calculate the mean and variance of the energy of each subsystem.

2. Answer the same questions as in Problem 1 with  $N_A = 20$ ,  $N_B = 20$ ,  $E_A = 100$ , and  $E_B = 20$ .

3. Answer the same questions as in Problem 1 with  $N_A = 20$ ,  $N_B = 40$ ,  $E_A = 100$ , and  $E_B = 20$ .

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4. If the two subsystems have equal numbers of particles, it is reasonable to conclude that the "hotter" system has higher energy. What is the probability that energy goes from the hotter to the colder system after the internal constraint has been removed?
5. Consider successively larger systems until you have satisfied yourself that you understand the qualitative behavior of the various quantities. Discuss your general qualitative conclusions.
6. Consider a special subsystem with only one particle,  $N_A = 1$ . Suppose that  $N_B = 5$ ,  $E_A = 0$ , and  $E_B = 12$ . If we assume that the subsystem  $A$  can exchange with the much larger system  $B$ , what is the probability that system  $A$  has energy  $E_A$ ? What is the probability that system  $A$  is in a particular microstate with energy  $n$  where  $n$  is an integer? The probability in this case is called the Boltzmann probability. Why is the form of this probability different than the probability that you found in the other problems?